HL Algebra Practice Questions
03/01/2007

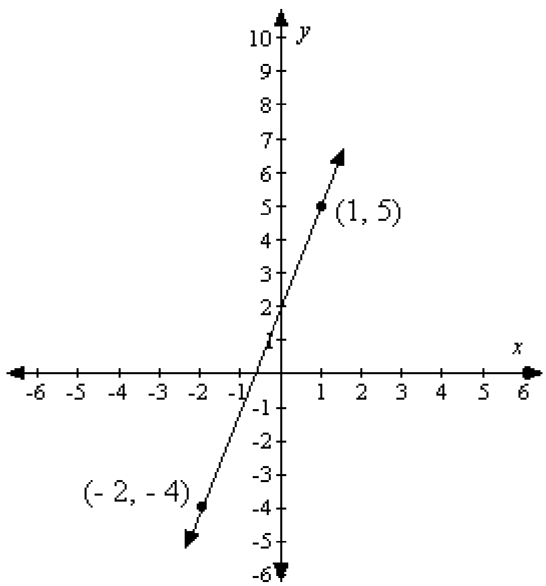
Student Name: ______________________________

Class: ______________________________

Date: ______________________________

Instructions: **Read each question carefully and select the correct answer.**

1. Calculate the slope of the line between the points (-2, - 4) and (1, 5).



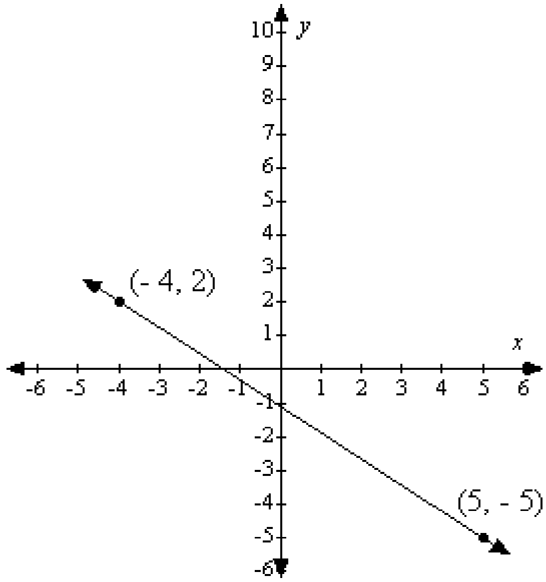
A. $-\frac{1}{3}$

B. 3

C. - 3

D. $\frac{1}{3}$

2. Calculate the slope of the line between the points (- 4, 2) and (5, - 5).



A. $-\frac{9}{7}$

B. $-\frac{7}{9}$

C. $\frac{7}{9}$

D. - 7

3. Which of the following relations is **not** a function?

A. {(11, - 9), (- 9, 11), (4, - 2), (- 13, 13), (10, 3)}

B. {(7, 3), (- 1, 9), (6, 15), (8, 0), (1, - 9)}

C. {(- 4, 3), (10, 14), (- 5, 1), (- 11, 3)}

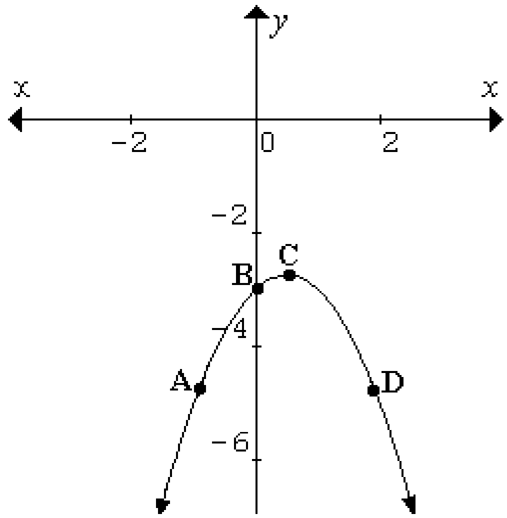
D. {(14, 0), (- 15, - 10), (9, 3), (14, - 1), (6, - 8)}

4. Which of the following points, if removed from the set, would make the set a function?

{(6, 8), (- 10, 8), (6, - 3), (14, 5), (- 4, - 9), (8, - 10), (10, - 8), (0, 9), (3, 9), (- 3, - 9)}

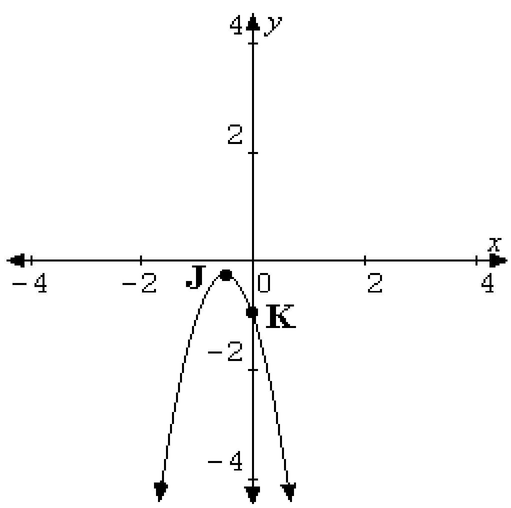
A. (- 10, 8)
B. (10, - 8)
C. (6, 8)
D. (- 4, - 9)

5. The following graph represents the equation $y = -x^2 + x - 3$. Choose the point(s) on the graph that would solve the equation $-x^2 + x - 3 = 0$.



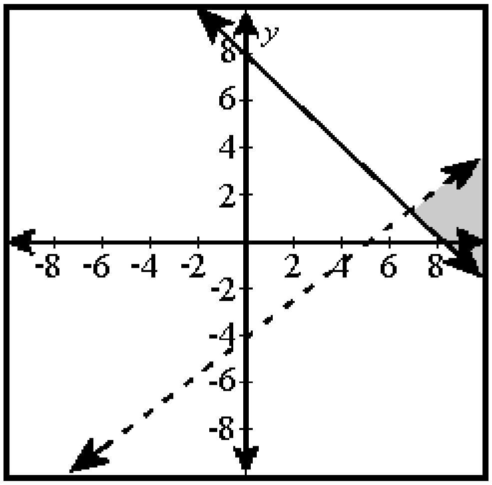
A. Point C
B. Point B
C. no real solutions
D. Points A and D

6. The following graph represents the equation $y = -3x^2 - 3x - 1$. Choose the point(s) on the graph that would solve the equation $-3x^2 - 3x - 1 = 0$.



A. There are no real number solutions.
B. All real numbers are solutions.
C. Point K
D. Point J

7. Choose the system of inequalities represented by the following graph.



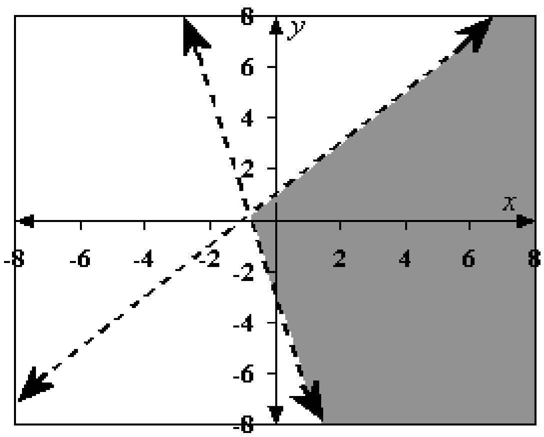
A. $y \geq \frac{3}{4}x - 4$
$y < -x + 8$

B. $y > \frac{3}{4}x - 4$
$y \leq -x + 8$

C. $y < \frac{3}{4}x - 4$
$y \geq -x + 8$

D. $y \leq \frac{3}{4}x - 4$
$y > -x + 8$

8. Choose the system of inequalities represented by the following graph.



A. $16x + 4y > -12$, $x - y > -1$

B. $16x - 4y < -12$, $x + y < -1$

C. $16x + 4y \geq -12$, $x - y \geq -1$

D. $16x - 4y > -12$, $x + y > -1$

9. For what value of x will the relation NOT be a function?

$$\{(\tfrac{x}{2} - 2x, 9), (\tfrac{9}{2}, x), (\tfrac{x}{2} + 6, 2)\}$$

A. x = 1/3
B. x = 9/2
C. x = -3
D. x = 9

10. Which option is a constant function?

A. y = 2/3x + 5
B. y = 5/8x + 1
C. y = -8
D. x = 3

11. Given the coordinates A(-3, 5) and C(7, -1), find the equation of the line.

A. $y = -3/5x + 3\ 1/5$
B. $y = -3/5x - 16/5$
C. $y = -3/5x$
D. $y = 19/5x$

12. Given the coordinates (2, 2) and (4, -1), find the equation of the line.

A. $3x + 2y = -1$
B. $x + 2y = 3$
C. $-x + 2y = 1$
D. $3x + 2y = 10$

13. Factor completely.

$8x^4 - 18x^2$

A. $8x^2(x^2 - 2)$

B. $2x^2(2x + 3)(2x - 3)$

C. $2x^2(4x^2 - 9)$

D. $x^2(8x^2 - 18)$

14. Factor completely.

$20x^2 - 36x - 35$

A. $(10x + 7)(2x - 5)$
B. $(10x - 7)(2x + 5)$
C. $(10x + 5)(2x - 7)$
D. $(10x - 5)(2x + 7)$

15. Find the discriminant of the quadratic equation and choose the statement which best describes the solution.

$12x^2 + 2x - 4 = 0$

A. The solutions are two distinct imaginary numbers (complex conjugates).
B. The solutions are two distinct real numbers.
C. There is only one real solution.
D. There are no real solutions.

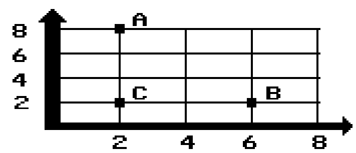
16. Find the discriminant of the quadratic equation and choose the statement which best describes the solution.

$$3x^2 + 12x + 12 = 0$$

A. There are no real solutions.
B. There is only one real solution.
C. The solutions are two distinct real numbers.
D. The solutions are two distinct imaginary numbers (complex conjugates).

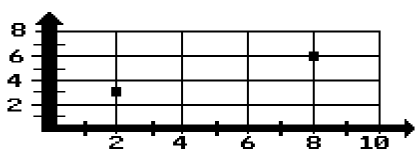
17. Which equation will give you the distance between point A and point B on a graph?

A. $AB = \sqrt{(2-6)^2 + (2-2)^2}$
B. $AB = \sqrt{52}$
C. 24
D. $6^2 + 2^2 = c^2$



A. A
B. B
C. C
D. D

18. Find the distance between point (2, 3) and point (8, 6) on the graph.



A. $3\sqrt{5}$

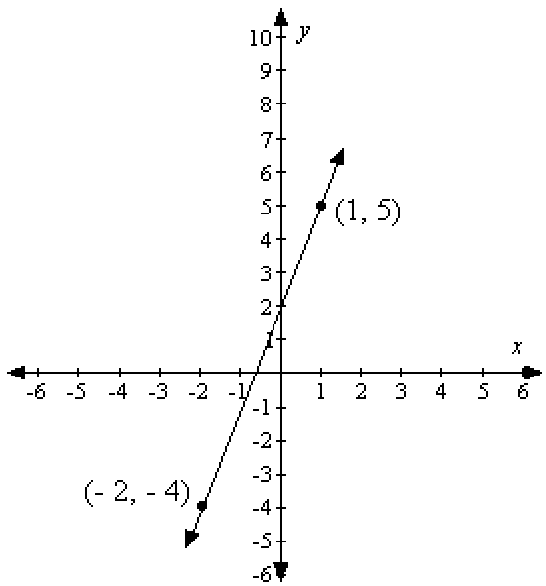
B. $\sqrt{3}$

C. $\sqrt{32}$

D. 5

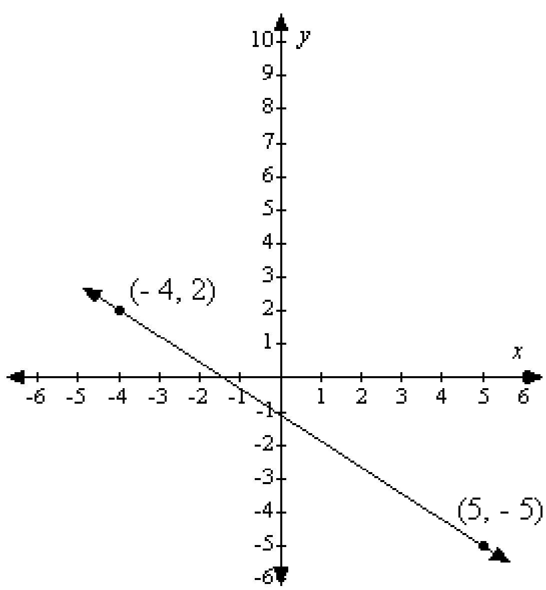
HL Algebra Practice
Answer Key
03/01/2007

1. Calculate the slope of the line between the points (-2, - 4) and (1, 5).



B. 3
Slope

2. Calculate the slope of the line between the points (- 4, 2) and (5, - 5).



B. $-\frac{7}{9}$
Slope

3. Which of the following relations is **not** a function?

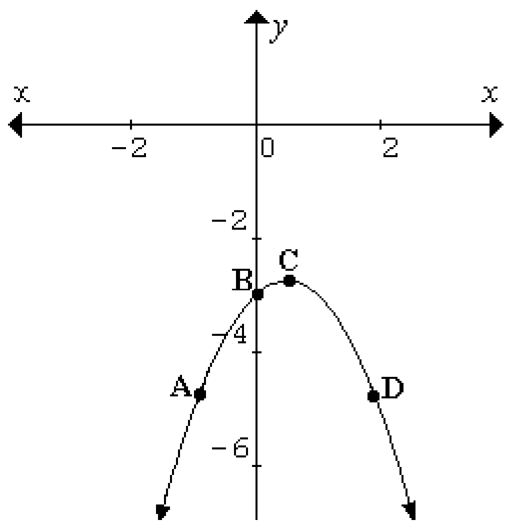
D. {(14, 0), (- 15, - 10), (9, 3), (14, - 1), (6, - 8)}
Functions/Relations - A

4. Which of the following points, if removed from the set, would make the set a function?

{(6, 8), (- 10, 8), (6, - 3), (14, 5), (- 4, - 9), (8, - 10), (10, - 8), (0, 9), (3, 9), (- 3, - 9)}

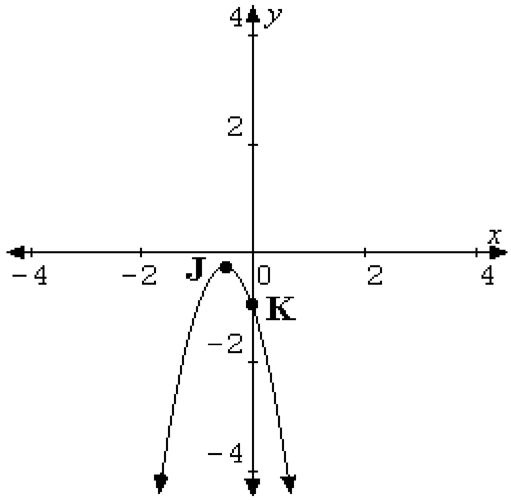
C. (6, 8)
Functions/Relations - A

5. The following graph represents the equation $y = -x^2 + x - 3$. Choose the point(s) on the graph that would solve the equation $-x^2 + x - 3 = 0$.



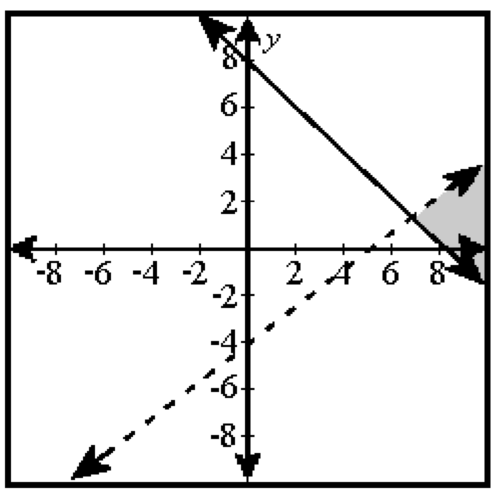
C. no real solutions
Solve Quadratic Equations by Graphing

6. The following graph represents the equation $y = -3x^2 - 3x - 1$. Choose the point(s) on the graph that would solve the equation $-3x^2 - 3x - 1 = 0$.



A. There are no real number solutions.
Solve Quadratic Equations by Graphing

7. Choose the system of inequalities represented by the following graph.

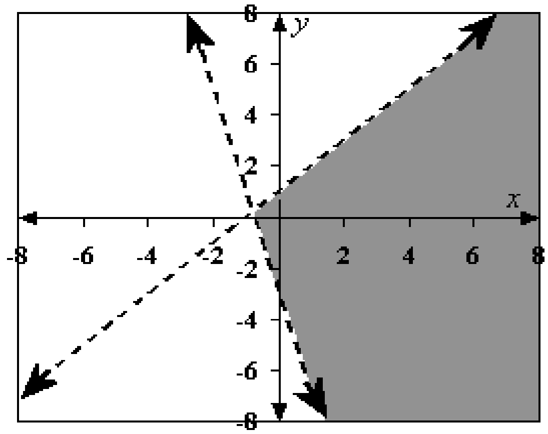


$y < \frac{3}{4}x - 4$

C. $y \geq -x + 8$

Graph Systems of Inequalities

8. Choose the system of inequalities represented by the following graph.



$$\begin{aligned}16x + 4y &> -12\\ x - y &> -1\end{aligned}$$

A.
Graph Systems of Inequalities

9. For what value of x will the relation NOT be a function?

$\{(\frac{x}{2} - 2x, 9), (\frac{9}{2}, x), (\frac{x}{2} + 6, 2)\}$

C. x = -3
Functions/Relations - B

10. Which option is a constant function?

C. y = -8
Functions/Relations - B

11. Given the coordinates A(-3, 5) and C(7, -1), find the equation of the line.

A. y = -3/5x + 3 1/5
Equations of a Line

12. Given the coordinates (2, 2) and (4, -1), find the equation of the line.

D. 3x + 2y = 10
Equations of a Line

13. Factor completely.

$8x^4 - 18x^2$

B. $2x^2(2x+3)(2x-3)$
Factoring

14. Factor completely.

$20x^2 - 36x - 35$

A. $(10x + 7)(2x - 5)$
Factoring

15. Find the discriminant of the quadratic equation and choose the statement which best describes the solution.

$12x^2 + 2x - 4 = 0$

B. The solutions are two distinct real numbers.
Quadratic Formula

16. Find the discriminant of the quadratic equation and choose the statement which best describes the solution.

$3x^2 + 12x + 12 = 0$

B. There is only one real solution.
Quadratic Formula

17. Which equation will give you the distance between point A and point B on a graph?

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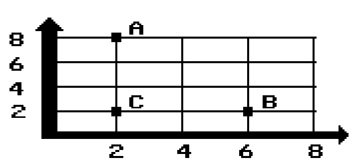
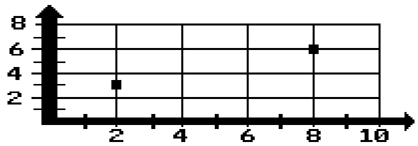


B. B
Distance Formula

18. Find the distance between point (2, 3) and point (8, 6) on the graph.



A. $3\sqrt{5}$

Distance Formula

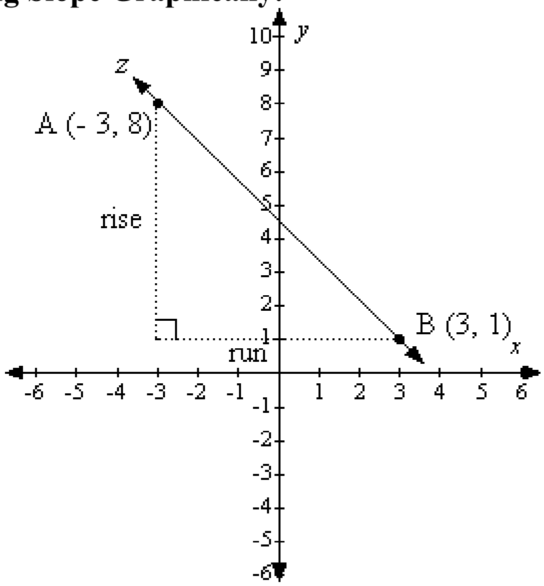
Study Guide

HL Algebra Practice
03/01/2007

Slope

Slope is the ratio of the vertical difference of two points on a line and the horizontal difference between the same two points. Slope is also defined as "rise over run" and is found by calculating the difference in the *y*-coordinates (rise) divided by the difference in the *x*-coordinates (run). Slope can be found either graphically or algebraically.

Finding Slope Graphically:



Two points are identified on line *z*: A (- 3, 8) and B (3, 1).
To find the slope of line *z*, which passes through points A and B, follow these steps.

Step 1: Start at the left-most point (it is possible to start at either point, but it is important to be consistent), which is point A (- 3, 8).
Step 2: Count up (positive) or down (negative) until level with the right-most point, point B (3, 1). The count will be 7 in the downward direction, or - 7. The *rise* is - 7.
Step 3: Count right (positive) or left (negative) until point B is reached. The count will be 6 to the right, or +6. The *run* is 6.
Step 4: Using the "rise over run" definition of slope, place - 7 on top of the fraction and 6 on the bottom of the fraction.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-7}{6} = -\frac{7}{6}$$

Finding Slope Algebraically:

To find the slope of line *z* algebraically, follow these steps.

Step 1: Use the formula for the slope of a line, where *m* is the variable that represents slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$x_1 = -3 \quad x_2 = 3$
$y_1 = 8 \quad y_2 = 1$

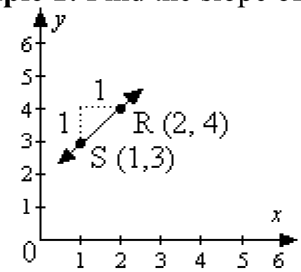
Step 2: Let (- 3, 8) be point (x_1, y_1), and (3, 1) be point (x_2, y_2).

Step 3: Substitute the given coordinate points into the slope formula and simplify the fraction.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-8}{3-(-3)} = -\frac{7}{6}$$

It does not matter which method (graphically or algebraically) is used for determining slope. Regardless, the slope of a line always remains the same.

Example 1: Find the slope of the line between Point R (2, 4) and Point S (1, 3).



Graphically:

Step 1: Start at the left-most point, which is point S (1, 3).

Step 2: Determine the rise (1).

Step 3: Determine the run (1).

Step 4: Using the "rise over run" definition of slope, place 1 on top of the fraction and 1 on the bottom of the fraction.

$$m = \frac{1}{1} = 1$$

Answer: $m = 1$

Algebraically:

(1)	(2)	(3)
$m = \frac{y_2 - y_1}{x_2 - x_1}$	$m = \frac{3 - 4}{1 - 2}$	$m = \frac{-1}{-1}$
		$m = 1$

Step 1: Write the formula.

Step 2: Substitute the given points into the formula. Let $(2, 4) = (x_1, y_1)$ and $(1, 3) = (x_2, y_2)$.

Step 3: Simplify the fraction.

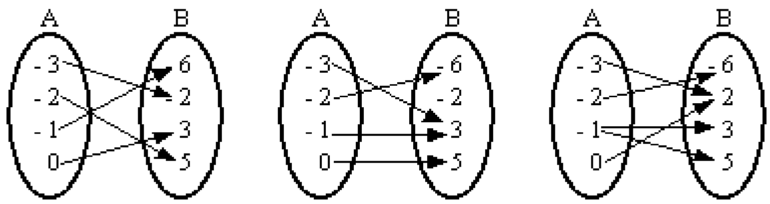
Answer: $m = 1$

An activity that can reinforce the concept of slope is to have students randomly plot two points on a coordinate system and then find the slope graphically. They can check their answers by substituting the two points into the slope formula.

Functions/Relations - A

A relation is a set of ordered pairs that represent a relationship between the elements of the two sets. A function is a special type of relation, where each element of the first set (x-values) corresponds to an unique element of the second set (y-values). The first set of numbers is commonly known as the input and the second set as the output. The input, or x-values, are entered into the equation. Once evaluated, the result is the output, or y-values. In other words, in order for a relation to be a function, for each x-value there can be no more than one

value of y. Some examples of relations are given below, with input values in A mapped to output values in B.



Relation 1 Relation 2 Relation 3

Relations 1 and 2 are functions, while relation 3 is not a function. The input value - 1 in relation 3 is matched to more than one output value (3 and 5), so the relation is not a function.

Example 1:
Which of the following relations is <u>not</u> a function?

(A) {(6, - 9), (12, 4), (- 10, - 3), (4, 12)}

(B) {(7, - 10), (4, 4), (- 7, 10), (11, - 5)}

(C) {(9, - 1), (- 12, - 1), (9, 4), (15, -11)}

(D) {(7, - 1), (9, -14), (13, - 5), (- 5, - 1)}

<u>Solution:</u>
If there is a value of x resulting in more than one value of y, the relation is not a function. This only occurs in the third set of numbers with (9, - 1) and (9, 4). Therefore, set C is not a function.

Answer: Set C is not a function.

Example 2:
Which of the following points, if removed from the set, would make the set a function?

{(- 4, 5), (4, - 5), (- 4, 4), (- 5, 5), (5, - 4)}

<u>Solution:</u>
The ordered pairs (- 4, 5) and (- 4, 4) have the same x values but different y values. Therefore, if either point is removed from the set, the remaining ordered pairs will represent a function.

Answer: Remove either (- 4, 5) or (- 4, 4).

Solve Quadratic Equations by Graphing

A <u>quadratic equation</u> is a function that contains polynomial expressions for which the highest power of the unknown variable is two.

Quadratic functions are written in the form:

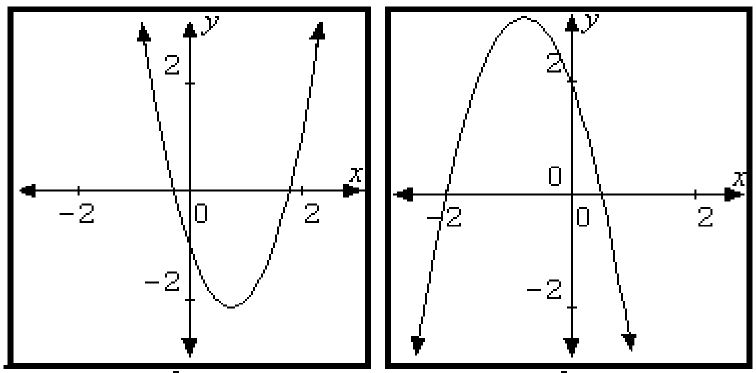
$$y = ax^2 + bx + c \text{ or } f(x) = ax^2 + bx + c$$

f(x) is read "f of x."

Below are a few examples of quadratic functions:

$y = x^2 + 3x - 4$ $\qquad$ $g(x) = -2x^2 + 6$ $\qquad$ $f(x) = 5x^2 - 2x$

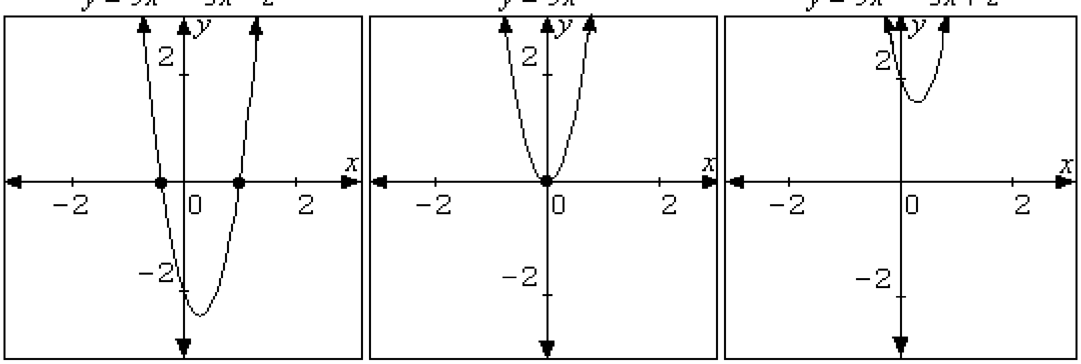
Graphs of quadratic functions are always in the shape of a <u>parabola</u>. Parabolas can open up or open down. Examples of each are shown below.



$y = 2x^2 - 3x - 1$ $\qquad$ $y = -2x^2 - 3x + 2$

Factoring, using the quadratic formula, and graphing are the three main methods for solving quadratic equations. This skill focuses on solving quadratic equations by graphing. To solve quadratic equations, it is necessary to find the values for *x* in which *y* equals zero. These values occur at the *x*-intercept(s), or the point(s) where the graph crosses the *x*-axis. The *x*-intercepts are of the form (*x*, 0), where the *y*-value equals zero. *X*-intercepts can occur at two points, one point, or no points.

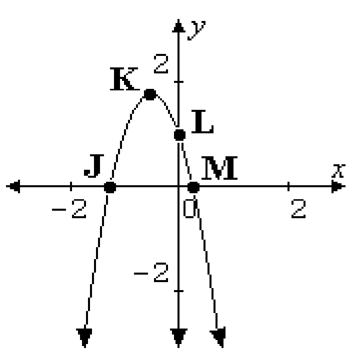
$y = 5x^2 - 3x - 2$ $\qquad$ $y = 5x^2$ $\qquad$ $y = 5x^2 - 3x + 2$



Two real number solutions $\qquad$ One real number solution $\qquad$ No real number solutions

Example 1:

The following graph represents the equation $y = -3x^2 - 3x + 1$. Choose the point(s) on the graph that would solve the equation $-3x^2 - 3x + 1 = 0$.



Solution:
The x-intercepts are the solutions to the quadratic equation. Therefore, points J and M would solve $- 3x^2 - 3x + 1 = 0$.

Answer: Points J and M.

Graph Systems of Inequalities
An inequality is a number sentence that uses *is greater than* or *is less than* symbols. For example, $6n < 4$ and $y \geq 2x - 3$ are inequalities.

When graphing an inequality, the student should mentally replace the inequality symbol with an equal sign in order to graph the inequality as an equation. Then use the table below to decide the type of line that should be used when drawing the graph.

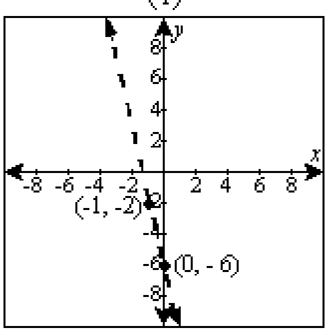
Symbol:	Definition:	Type of Line:
$>$	is greater than	Dashed
$<$	is less than	Dashed
$\geq$	is greater than or equal to	Solid
$\leq$	is less than or equal to	Solid

A dashed line tells the reader that the values on the line ARE NOT included in the inequality. A solid line tells the reader that the values on the line ARE included in the inequality.

Example 1:
Graph the inequality.

$y > - 4x - 6$

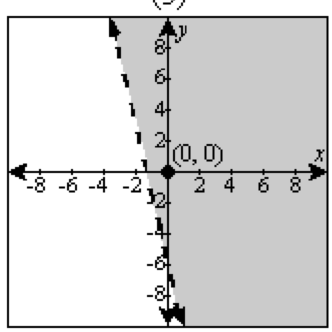
(1)



(2)

$y > -4x - 6$
$0 > -4(0) - 6$
$0 > -6$
This is TRUE!

(3)

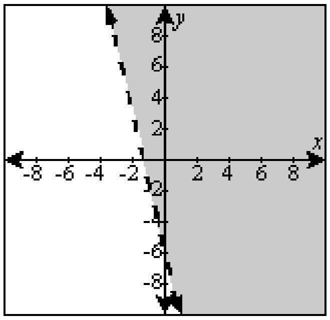


Step 1: Graph the line that is represented by the inequality. (Remember to mentally replace the > with =.) This equation is given in $y = mx + b$ form (slope-intercept form), where m is the slope and b is the y-intercept. Plot the y-intercept, (0, - 6), then use the slope, - 4, to move up 4 units and to the left 1 unit. The *is greater than* symbol (>) is used, refer to the chart above to see that this symbol requires a dashed line. Connect the two points using a dashed line.
Step 2: Choose a test point (that is not on the line) to determine which side of the line should be shaded. The most common test point to use is (0, 0), but it does not matter what point is used. Substitute the test point into the inequality and simplify.

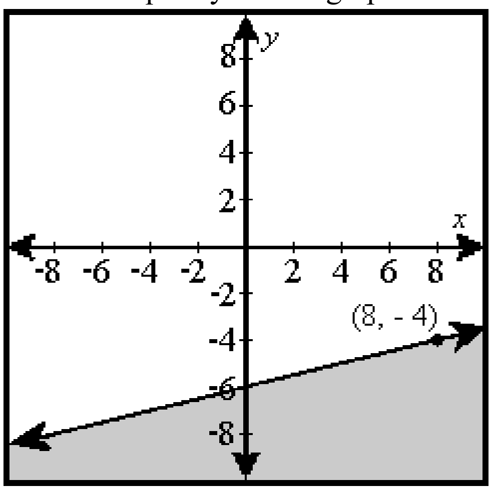
- **If the test point makes the inequality true, shade the side of the line that includes the test point.**
- **If the test point makes the inequality false, shade the side of the line that does not include the**

test point.
In this case, the test point makes the inequality true.
Step 3: Since the test point makes the inequality true, shade the side of the dotted line that includes the point (0, 0).

Answer:



Example 2:
Determine the correct inequality for the graph below.



(1) $y = \frac{1}{4}x - 6$

(2) $y \le \frac{1}{4}x - 6$ OR $y \ge \frac{1}{4}x - 6$

(3)

$y \le \frac{1}{4}x - 6$	OR	$y \ge \frac{1}{4}x - 6$
$-8 \le \frac{1}{4}(0) - 6$		$-8 \ge \frac{1}{4}(0) - 6$
$-8 \le -6$		$-8 \ge -6$
TRUE		FALSE

Step 1: Determine the equation of the line. In this case, the *y*-intercept is at (0, - 6) and the slope appears to be *up 1, over 4* (or ?), as can be seen by points at (8, - 4) and (4, - 5). Therefore, the equation of the boundary line is $y = (?)x - 6$.
Step 2: Use the table on page 1 to determine which type of inequality symbol to use (<, >, ≤, or ≥). The line on the graph is solid, so the ≤ or ≥ symbol must be used.
Step 3: Choose a test point from the shaded side of the line, and substitute it into each inequality to determine which of the two inequalities is correct. A good test point to use is (0, - 8), since (0, - 8) is included in the shaded area of the graph. Since $y \le (?)x - 6$ is true when (0, - 8) is used as the test point,

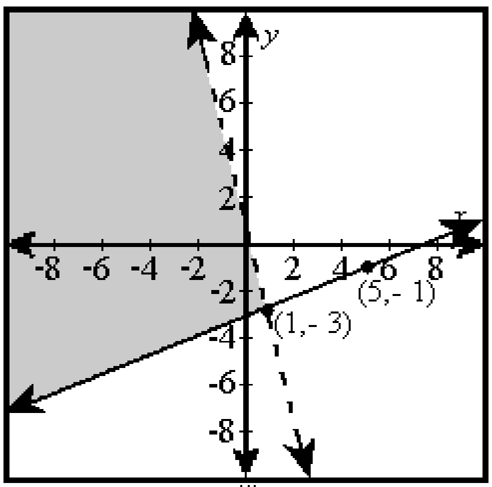
it is the correct inequality.

Answer: $y \le \frac{1}{4}x - 6$

Systems of Inequalities:

When graphing a system of inequalities, the process is very similar. A <u>system of inequalities</u> is two or more inequalities. The main difference is that the final solution is **the area where the shaded regions overlap**.

Example 3:
Choose the system of inequalities represented by the following graph.



(1)

$$y = \frac{2}{5}x - 3$$
$$y = -4x + 1$$

(2)

$y \le \frac{2}{5}x - 3$ OR $y \ge \frac{2}{5}x - 3$ $\quad$ $y < -4x + 1$ OR $y > -4x + 1$

(3)

$y \le \frac{2}{5}x - 3$ OR	$y \ge \frac{2}{5}x - 3$	$y < -4x + 1$ OR	$y > -4x + 1$
$0 \le \frac{2}{5}(-2) - 3$	$0 \ge \frac{2}{5}(-2) - 3$	$0 < -4(-2) + 1$	$0 > -4(-2) + 1$
$0 \le -3\frac{4}{5}$	$0 \ge -3\frac{4}{5}$	$0 < 9$	$0 > 9$
FALSE	TRUE	TRUE	FALSE

<u>Step 1</u>: Determine the equations of the boundary lines. The solid line has a *y*-intercept of - 3 and a slope of 2/5. The dashed line has a *y*-intercept of 1 and a slope of - 4. The equations of the boundary lines are $y = (2/5)x - 3$ (solid line) and $y = -4x + 1$ (dashed line).
<u>Step 2</u>: Use the chart on page 1 to determine which inequality symbols to use.
<u>Step 3</u>: Choose a test point from the shaded region of the graph and substitute it into each of the inequalities. A good test point to use is (-2, 0). Since (-2, 0) is in the shaded area of the graph, the inequalities that are true when (-2, 0) is substituted are the correct inequalities.

$y \geq \frac{2}{5}x - 3$

Answer: $y < -4x + 1$

Systems of Inequalities That Are Not Solved For *y*:

Sometimes systems of equations or inequalities are presented in a form other than y = *mx* + *b*. If an inequality is not in this form, the student should first solve the inequality for *y* in order to make the graphing process easier.

Remember, to solve an inequality for *y*, use inverse operations to isolate the variable and be sure to follow inequality sign rules when multiplying or dividing.

Rules:

•When multiplying or dividing both sides of an inequality by a <u>positive</u> number, leave the inequality sign as is.

•When multiplying or dividing both sides of an inequality by a <u>negative</u> number, <u>reverse</u> the direction of the inequality sign.

•When adding or subtracting both sides of an inequality by a <u>positive or negative</u> number, leave the inequality sign as is.

Example 4:

Rewrite the following system of inequalities in order to solve for *y*. For this example you do not need to actually graph the system.

$8x - 4y > 24$

$5 \leq 6y + x$

(1)

$-4y > -8x + 24 \qquad 5 - x \leq 6y$

(2)

$\frac{-4}{-4}y > \frac{-8x}{-4} + \frac{24}{-4} \qquad \frac{5}{6} - \frac{1}{6}x \leq \frac{6y}{6}$

(3)

$y < 2x - 6 \qquad \frac{5}{6} - \frac{1}{6}x \leq y$

<u>Step 1</u>: Isolate the *y*-term by adding or subtracting the *x*-term.

<u>Step 2</u>: In order to isolate the variable in the first inequality, it is necessary to divide both sides of the inequality by a - 4. In the second inequality, both sides need to be divided by 6.

<u>Step 3</u>: Simplify each term in the inequality. Remember, since the first inequality was divided by a negative number, the direction of the inequality sign must be changed. In the second inequality, the sign remains the same since the division was by a positive number.

nswer:

$y < 2x - 6$

$\frac{5}{6} - \frac{1}{6}x \leq y$

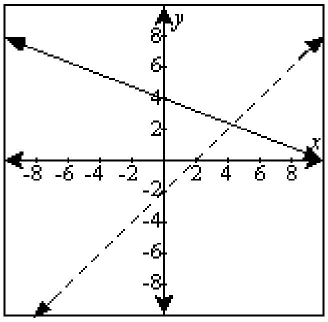
Example 5:

Graph the solution to the following system of inequalities.

$y \leq -\frac{1}{3}x + 4$

$y > x - 2$

(1)



(2)

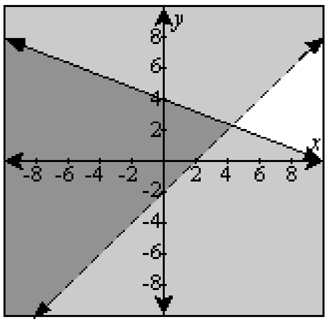
$$y \le -\frac{1}{3}x + 4 \qquad y > x - 2$$

$$0 \le -\frac{1}{3}(0) + 4 \qquad 0 > 0 - 2$$

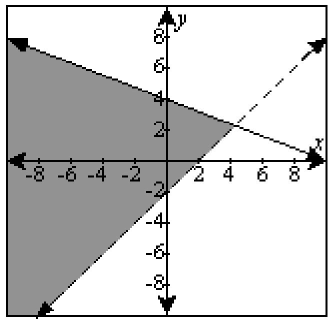
$$0 \le 4 \qquad 0 > -2$$

TRUE TRUE

(3)

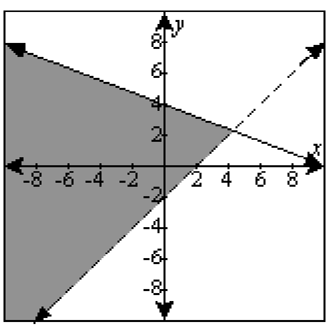


(4)



Step 1: Graph the lines that are represented by the inequalities. The y-intercept of the top line is 4 and the slope is -1/3. The y-intercept of the second line is -2 and the slope is 1. Use the chart on page 1 to determine whether the lines should be dashed or solid. The line with the $\le$ symbol should be solid, and the line with the > symbol should be dashed.
Step 2: Choose a test point, and substitute it into both inequalities to determine which direction to shade. It does not matter which point is used as a test point as long as the correct side of the line is shaded. A good point to use is (0, 0).
Step 3: Since the point (0, 0) makes both inequalities true, shade each inequality on the side of the line that contains (0, 0).
Step 4: The solution to the two inequalities is the region of the graph where the shading overlaps.

Answer:



Functions/Relations - B
A relation can be expressed as a set of ordered pairs such as $\{(3, 1), (2, 3), (-1, -2)\}$. A function is a special type of relation.

There are a few ways to determine if a relation is a function. One way is to look at all of the ordered pairs of a relation. If no two of these ordered pairs have the same x - term (abscissa), then the relation is

a function. Another method involves graphing the ordered pairs of a relation on a coordinate graph. If no vertical line crosses this graph at more than one point, then the relation is a function.

A linear function is a function whose graph is a line or subset of a line which is not vertical. A special type of linear function is a constant function, whose graph is a horizontal line or subset of a horizontal line.

Using these guidelines, find the value for q that will make the following relation not a function: $\{(16, 3), (q^2, 7)\}$.

The answer is 4 because if g = 4, then the second ordered pair would be (16, 7). The two ordered pairs would have the same x - term, and for that reason the relation would not be a function.

Equations of a Line
Every line on any coordinate graph has a corresponding equation which describes every point on the line. Every linear equation (equation of the line) contains a slope. The slope of a line is the same between any two points on the line.

Before you can find the equation of a line, you must first be able to find the slope of a line when given two coordinate points on the line. These two points are named: (x_1, y_1) and (x_2, y_2). The formula for the slope of a line follows.

$$\text{Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1: Find the slope of the line between Point R (2, 4) and Point S (1, 3).

(1) $m = \frac{3-4}{1-2}$ **(2)** $m = \frac{-1}{-1}$

$m = 1$

<u>Step 1</u>: Substitute the given coordinate points into the formula.
<u>Step 2</u>: Simplify the fraction.

Answer: The slope of the line is 1.

The Point-Slope form for the equation of a line:

$$y - y_1 = m(x - x_1)$$

Example 2: Use the following points to find the equation of the line.
Point T (7, -3)
Point U (-4, 6)

(1)

$$m = \frac{6--3}{-4-7}$$

$$m = \frac{6+3}{-4-7}$$

$$m = \frac{9}{-11} = -\frac{9}{11}$$

(2)

$$y--3 = -\frac{9}{11}(x-7)$$

(3)

$$y+3 = -\frac{9}{11}x + \frac{63}{11}$$

(4)

$$\begin{array}{rl} y+3 = & -\frac{9}{11}x + \frac{63}{11} \\ -3 & \quad\quad -3 \\ \hline y = & -\frac{9}{11}x + \frac{30}{11} \end{array}$$

Step 1: Solve for the slope of the line between Point T and Point U.
Step 2: Use one of the coordinate points and the slope and substitute them into the Point-Slope form for the equation of a line.
Step 3: Simplify both sides of the equation.
Step 4: Subtract 3 from both sides of the equation.

The equation of the line that passes through (7, -3) and (-4, 6) is y = -9/11x + 30/11.

Factoring

Consider the following equation:

3 x 4 = 12

The numbers 3 and 4 are said to be factors of the number 12. This concept of factoring is not reserved for numbers, but may be extended to polynomials as well.

Factoring is the breaking up of quantities into products of their component factors. One way to think of factoring is as the opposite or inverse of multiplying.

A polynomial is a term or sum of terms. Each term is either a number or a product of a number and one or more variables.

A monomial is a polynomial with one term.

A binomial is a polynomial with two terms.

A trinomial is a polynomial with three terms.

Consider the following polynomial:

$$4y^3 + 16y^2 - 20y$$

A typical question on factoring will include a polynomial like the one above. Notice that 4y is a common factor of each term of the polynomial.

Step 1: Factor out the 4y by dividing each term of the trinomial by 4y.

$$\frac{4y^3}{4y} + \frac{16y^2}{4y} - \frac{20y}{4y}$$

$$4y\left(y^2 + 4y - 5\right)$$

Step 2: The trinomial in parentheses can be factored further. Since the coefficient of the "y squared"

term is equal to 1, focus on the last term, in this case, -5. If factors of -5 can be found that ADD up to the coefficient of the middle term (in this case, 4) the trinomial can be factored. Two factors of -5 are 5 and -1, and when ADDED together the result is equal to the coefficient of the middle term, 4. Notice how these numbers are put together to construct the fully factored trinomial:

$(4y)(y + 5)(y - 1)$

To check the result, use the rules for multiplying polynomials and you should have the original polynomial when finished.

Sometimes factoring must be done by grouping. The polynomial given below may appear impossible to factor at first, but if you examine the steps you will see a method to use with polynomials of this type.

$$15x^2 + 20xy + 18nx + 24ny$$
$$\text{Step 1}: 5x(3x + 4y) + 18nx + 24ny$$
$$\text{Step 2}: 5x(3x + 4y) + 6n(3x + 4y)$$
$$\text{Step 3}: (5x + 6n)(3x + 4y)$$

Step 1: $(5x)$ is a common term to both $15x$ and $20xy$. Factor it out of only those two terms.
Step 2: $(6n)$ is a common term of both $18nx$ and $24ny$. Factor it out of only those two terms.
Step 3: Notice that the quantity $(3x + 4y)$ is a common factor of $5x$ and $6n$. The expression is rewritten to indicate this, and the polynomial is completely factored.

A special type of polynomial expresses the difference of two perfect squares. Polynomials of this type are factored easily once the rule is remembered.

$$x^2 - 36$$
$$(x + 6)(x - 6)$$

Since each term in the polynomial is a perfect square, the square root of each term (in this case x and 6 respectively) will be used in the following way. The original polynomial is factored as $(x + 6)(x - 6)$. Notice that if these terms are multiplied together, the original polynomial is formed. Polynomials that are in the "difference of squares" form may always be factored as the sum of the square roots times the difference of the square roots.

Quadratic Formula
A quadratic equation is a polynomial equation in which the highest power of the unknown variable is two.

An example of a quadratic equation is below.

$x^2 + 6x - 91 = 0.$

The format of a quadratic equation is $ax^2 + bx + c = 0.$ Quadratic equations can be solved by factoring, graphing, or by using the quadratic formula. The quadratic formula is as follows:

$$\boxed{\begin{array}{l}\text{Quadratic Formula}\\ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}\end{array}}$$

It can be found in any algebra textbook. This formula should be memorized.

To apply the formula to a quadratic equation, use the quadratic equation format given above as a guideline.

Example 1: Solve the quadratic equation.

$$x^2 + 6x - 91 = 0$$

(1) **(2)** **(3)**

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-91)}}{2(1)} \qquad x = \frac{-6 \pm \sqrt{400}}{2} \qquad x = \frac{-6 \pm 20}{2}$$

(4)

$$x = \frac{-6 + 20}{2} \text{ and } x = \frac{-6 - 20}{2}$$

$$x = \frac{14}{2} \qquad x = \frac{-26}{2}$$

$$x = 7 \text{ and } x = -13$$

Step 1: Determine the values of a, b, and c and substitute them into the quadratic formula. a = 1, b = 6, and c = -91
Step 2: Determine the value under the radical symbol. 6 squared is 36 and -91 times -4 equals 364. 36 + 364 = 400
Step 3: The square root of 400 is 20 (20 x 20 = 400).
Step 4: Split the remaining problem into two problems: (-6 + 20) ÷ 2 and (-6 - 20) ÷ 2 and solve the two problems.

The answers are x = 7 and x = -13.

Example 2: Solve the quadratic equation.

$$5x^2 + 2x + 8 = 4x^2 - 2x + 4$$

$$\text{(1) } 5x^2 + 2x + 8 = 4x^2 - 2x + 4$$

$$\text{(2) } \begin{array}{r} 5x^2 + 2x + 8 = 4x^2 - 2x + 4 \\ -4x^2 \qquad\qquad -4x^2 \\ \hline x^2 + 2x + 8 = -2x + 4 \\ +2x \qquad +2x \\ \hline x^2 + 4x + 8 = \ 4 \\ -4 \ \ -4 \\ \hline x^2 + 4x + 4 = 0 \end{array}$$

$$\text{(3) } a = 1,\ b = 4,\ c = 4$$

$$\text{(4) } x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(4)}}{2(1)}$$

$$\text{(5) } x = \frac{-4 \pm \sqrt{16 - 16}}{2} = \frac{-4 \pm 0}{2}$$

$$\text{(6) } x = \frac{-4}{2} = -2$$

Step 1: Write the equation.

Step 2: Subtract $4x^2$ from both sides of the equation. Then, add $2x$ to both sides of the equation. Finally, subtract 4 from both sides of the equation. This will put the equation in standard form.

Step 3: Determine the values of a, b, and c.

Step 4: Substitute the values of a, b, and c into the quadratic formula.

Step 5: Determine the value under the radical sign. The square root of 0 is 0.

Step 6: Solve for x.

Answer: x = -2

The discriminant is the portion of the quadratic equation under the radical sign $b^2 - 4ac$. The discriminant properties below will give you vital information about quadratic equations.

1. If the discriminant is a perfect square, then the quadratic equation can be factored.
2. If the discriminant is greater than 0, then the equation has two real solutions.
3. If the discriminant is less than 0, then the equation has no real solutions.
4. If the discriminant is equal to 0, then the equation has one real solution.

Example 3: How many solutions does the following quadratic equation have?

$$3x^2 + 5x - 12 = 0$$

$$\text{(1) } a = 3,\ b = 5,\ c = -12$$

$$\text{(2) } (5)^2 - 4(3)(-12)$$

$$\text{(3) } 25 + 144 = 169$$

Step 1: Determine the values of a, b, and c.

Step 2: Substitute the values for a, b, and c into $b^2 - 4ac$. Step 3: Simplify the discriminant.

Since the discriminant is greater than zero, there are two real solutions.

If the discriminant is a perfect square, then the solutions are rational.
If the discriminant is not a perfect square, then the solutions are irrational.

Distance Formula

The distance formula can be used to find the distance between two points on a coordinate plane.

If you need to find the distance, d, between Point A and Point B on a coordinate plane, you can use the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Some distance problems can be solved by using the Pythagorean theorem. Pythagoras was a Greek mathematician, philosopher, and theologian who lived around 580 to 500 B.C. He proved the universal validity of a theorem later called the Pythagorean theorem.

The Distance Formula can be applied to find the distance between any two points.

Example: On a coordinate plane, point D is (3, 7), and point E is (6, 2). How far is point D from point E?

(1) $DE = \sqrt{(6-3)^2 + (2-7)^2}$

(2) $DE = \sqrt{3^2 + (-5)^2}$

(3) $DE = \sqrt{9+25}$

(4) $DE = \sqrt{34}$

Step 1: Substitute the values of the variables into the distance formula.

$x_1 = 3 \quad y_1 = 7$

$x_2 = 6 \quad y_2 = 2$ Step 2: Simplify (6 - 3) and (2 - 7).

Step 3: Square the two results of step 2.

Step 4: Simplify the radicand (the expression under the radical sign).

Answer: $\sqrt{34}$